Experiment 29. Galactic dynamics

Updated RWH 21-Jul-07

1 Introduction

The Small Radio Telescope (SRT) is designed for spectral line observations at the neutral Hydrogen (HI) 1420.4 MHz frequency (21 cm wavelength). This frequency corresponds to the hyperfine spin-flip transition in Hydrogen: the energy difference between an excited state of Hydrogen, when the spin magnetic moments of the proton and electron are parallel, and the ground state, when the spins are opposed (see Figure 29-1). Measurements of this line can be used to map the distribution and relative velocity of Hydrogen in our galaxy. With this information we can determine the rotation curve of the galaxy, infer its mass and see structures such as its spiral arms.

The specifications of the telescope are given in Table 1. The telescope is fully computer controlled with motorised drives enabling it to point to and track a source on almost all the visible sky. The telescope was developed by Massachusetts Institute of Technology's (MIT) Haystack Observatory as an introductory radio astronomy experiment.

The following websites are useful references for this experiment:
http://www.haystack.mit.edu/edu/undergrad/srt/
http://www.haystack.mit.edu/edu/undergrad/materials/RA_tutorial.html.

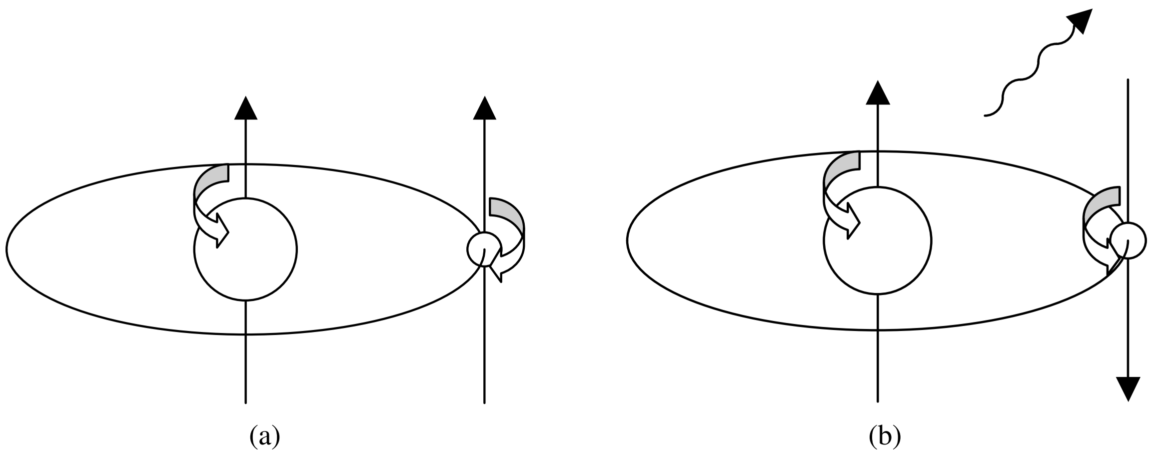

Fig. 29-1 : The Hydrogen hyperfine spin-flip transition: (a) excited parallel-spins state (note that the vertical arrows refer to the magnetic moments of the proton and electron), (b) ground antiparallel-spins state. The energy difference $\Delta E = h\nu$ between these states corresponds to a photon of frequency 1420.4 MHz or wavelength 21 cm.

Table 29-1 : Specifications of the Small Radio Telescope.

Aperture	3.0 m
Frequency Range	1400–1440 MHz or $\sim \pm 4000$ km s^{-1}
Frequency Resolution (3dB Bandwidth)	40 kHz or ~ 8 km s^{-1}
System Temperature	150 K
Sensitivity (0.1 s integration/40 kHz step)	~ 3 K or ~ 2000 Jy
Aperture Efficiency	$\sim 50\%$
Pointing Accuracy	1°
Travel Limits	Azimuth $\sim 273°$–$360°$, $0°$–$85°$
	Elevation $\sim 3.5°$–$175°$

2 SRT Operating Program

See the SRT Manual for details of the operating program. The program is started by double-clicking on the *Small Radio Telescope* icon on the desktop.

3 Experimental procedure

3.1 Pointing Accuracy

Ensure the SRT controller is on, connected to the computer's serial port and the SRT program is running. When the SRT program is started the telescope drives to its homing *stow* position (pointing West)[1]. This position is at one of the limits of the telescope's range of movements. The telescope can only be positioned relatively, so it needs to drive to this homing position in order to determine where the telescope is pointing. It takes a few minutes for the telescope to move from its *parked* position (pointing straight up—a safe position in high winds) to its homing position. The telescope is located on the roof of the Annex and can be seen from the car park between the two physics buildings.

1. After the telescope has arrived at its stow position (indicated by the cross in the SRT Program's Sky Map changing from yellow to red), click on the *Sun* source in the Sky Map to make the telescope slew to and track the Sun.

2. To map out the emission around the Sun, click the *npoint* button in the command toolbar. This records the signal strength over a 5×5 grid of points around the Sun. When the measurements are complete the results are displayed in a false colour contour plot to the left of the accumulated spectrum plot[2]. If the telescope is accurately pointing at and tracking the Sun, the image of the emission from the Sun should be in the centre of the npoint plot. If it is not, it probably means that the computer's time is wrong. Consult a tutor before making any adjustments.

3. Print a copy of your npoint plot of the Sun for your logbook. You can copy the SRT program window using Window's screen capture function. Make sure the SRT program is the active window and use the `<ALT>`-`<PRINT SCREEN>` key combination to copy the active window to the clipboard. Open Microsoft Word, paste the the clipboard contents into a new document and print a copy of the document for your logbook.

Question: Can you explain why the 'image' of the Sun is not quite circular? (Hint: how long did it take to do the 25-point scan?)

[1]This stow position is used in northern USA where there is a risk of the telescope filling with snow if it is left pointing upwards. In Sydney wind is a greater risk and the safest place to park the telescope is pointing straight up.

[2]If the npoint plot disappears, as happens, for example, when the program window is minimised, it can be redrawn by clicking within the area where the npoint plot is normally displayed.

3.2 Antenna Beamwidth and Resolution

The beamwidth of a radiotelescope antenna indicates the range of angles over which the telescope is sensitive to radiation at a given wavelength. It is normally given as the angular range where the telescope's sensitivity is half its maximum value: the full-width half maximum (FWHM) beamwidth. This 'half-maximum' point is also referred to as the -3 dB point (i.e. $10\log_{10}(1/2) \approx -3$). The beamwidth is also often referred to as the resolution of a radio telescope antenna, as the antenna cannot resolve (or distinguish between) angular structures smaller than the beamwidth.

According to diffraction theory the FWHM beamwidth, θ_{FWHM}, of a prime-focus parabolic reflector is given by

$$\theta_{\mathrm{FWHM}} = 1.22\lambda/D \tag{1}$$

where D is the diameter and λ is the operating wavelength of the telescope and θ_{FWHM} is in radians.

1. Determine θ_{FWHM} for the SRT at 1420 MHz, given the dish diameter stated by the US manufacturer is $D = 3.0\,\mathrm{m}$.
2. The angular size of the Sun at 1420 MHz is $\sim 0.75°$. Does the *npoint* plot show the structure of emission from the Sun or does it simply map out the antenna response?

C1 ▷

4 Galactic Rotation Curve

The flatness of the Milky Way suggests matter in the Galaxy rotates about an axis normal to the Galactic plane. Observations of the motions of stars and interstellar gas in the Galaxy have confirmed this rotation and shown it to vary with distance from the Galactic Centre. A galactic rotation curve is a plot of the average tangential velocity V of matter rotating around the centre of a galaxy as a function of radius R from the centre of the galaxy. One of the most effective methods of determining a galactic rotation curve is from measurements of the Doppler-shifted neutral Hydrogen 1420.4 MHz emission line, as these frequencies, unlike optical frequencies, are not strongly absorbed or scattered by interstellar dust. These measurements can therefore be used to map the structure and gas motions in the Milky Way. The SRT is sufficiently sensitive to detect emission from neutral Hydrogen on the opposite edge of the Milky Way and even from our neighbouring galaxies, the Magellanic Clouds.

The position of a cloud of neutral Hydrogen cannot be determined directly, but there is a method that can be used to infer the velocity V as a function of Galactic radius R. Assume the angular velocity of rotation ω about the Galactic Centre decreases with increasing distance R from the Galactic Centre. Then, as shown in Figure 29-2, for Galactic longitudes $0° < \ell < 90°$ in the plane of the Galaxy (Galactic latitude $b = 0°$), the maximum velocity along the line of sight is obtained at the point where the line of sight is tangent to a circle centred at the Galactic Centre. For Galactic longitudes $270° < \ell < 360°$ (or $-90° < \ell < 0°$) this point will correspond to maximum *negative* velocity. For longitudes $90° < \ell < 270°$, no such point exists. Using uppercase letters to denote variables relative to the Galactic Centre and lowercase letters for variables relative to our position, this tangent point is at

$$R = R_0 \sin \ell \tag{2}$$

(or $r = R_0 \cos \ell$) and has velocity along the line of sight

$$v_{\max} = R(\omega - \omega_0), \tag{3}$$

where R_0 is the distance and ω_0 is the angular velocity of our Local Standard of Rest (LSR) about the Galactic Centre. The velocity of the LSR about the Galactic Centre, $V_0 = 220\ \mathrm{km\,s^{-1}}$ toward $\ell = 90°$, $b = 0°$, is the average velocity of our Sun and its nearest ~ 100 neighbours. The distance of the LSR from the Galactic Centre is 8.5 kpc (kiloparsecs) or 28,000 lyr (light years) or 2.6×10^{17} km. Our Sun's velocity relative to the LSR is about $20\ \mathrm{km\,s^{-1}}$ toward $\ell = 56°$, $b = 23°$. The behaviour of the radial velocity relative to the LSR of other points along the line of sight as a function of radial distance from the LSR is shown schematically in Figure 29-3.

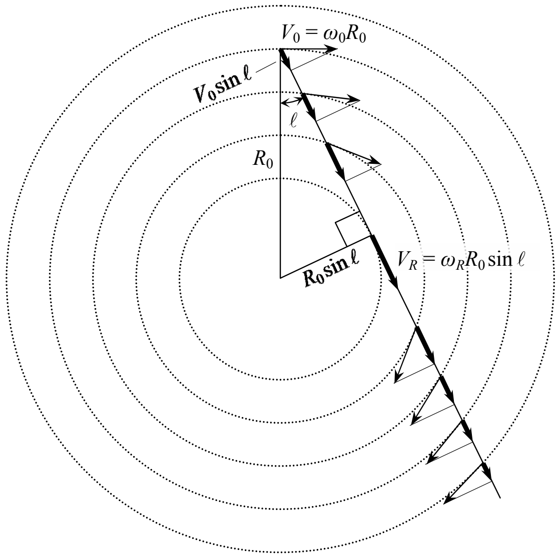

Fig. 29-2 : Geometry of velocity components for observations in the Galactic Plane at longitudes $0° < \ell < 90°$, accessible from the northern hemisphere.

If neutral Hydrogen is distributed throughout the Galaxy, the signal received by a radio telescope will contain contributions from emissions from neutral Hydrogen at each point along the line of sight through the Galaxy, as shown in Figure 29-4. More correctly, all lines of sight over which the telescope is sensitive (i.e. within the beamwidth of the telescope) should be considered. If the Hydrogen is moving relative to us and its motion has a component along our line of sight, we will observe its 1420.4 MHz emission line Doppler shifted to a different frequency. The radial velocity along the line of sight v ($\mathrm{km\,s^{-1}}$) is related to the measured Doppler frequency f (MHz) by

$$v = \frac{(1420.406 - f)c}{1420.406} - v_{\mathrm{LSR}}, \tag{4}$$

where $c = 299792\ \mathrm{km\,s^{-1}}$ is the speed of light and v_{LSR} is our velocity along the line of sight relative to the LSR. Note that the determination of v_{LSR} must account for not only our Sun's velocity relative to the LSR, but also for the motion of the Earth around the Sun and the rotation of the Earth at our observing position. Fortunately, the SRT software calculates v_{LSR} for us.

The amplitude of the spectrum at a given frequency is directly related to the amount of gas moving at the corresponding relative velocity along the line of sight. A greater concentration of gas moving at a particular relative velocity will have a higher amplitude signal. Note that clouds of gas on the same orbit radius will have equal velocities along the line of sight, so it is not possible to determine how much each one contributes to the total signal strength at that velocity. The only unambiguous signal is that corresponding to the maximum velocity (or minimum velocity for sightlines in the fourth quadrant). Rearranging (3), and noting that $V(R) = \omega R$ and $V_0 = 220\ \mathrm{km\,s^{-1}}$, gives

$$V(R) = v_{\max} + V_0 \sin \ell. \tag{5}$$

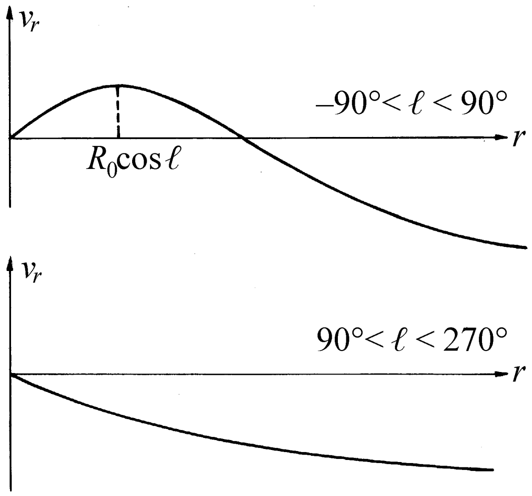

Fig. 29-3 : Velocity relative to the local standard of rest v_r as a function of distance r from the local standard of rest for lines of sight of varying Galactic longitude in the Galactic plane.

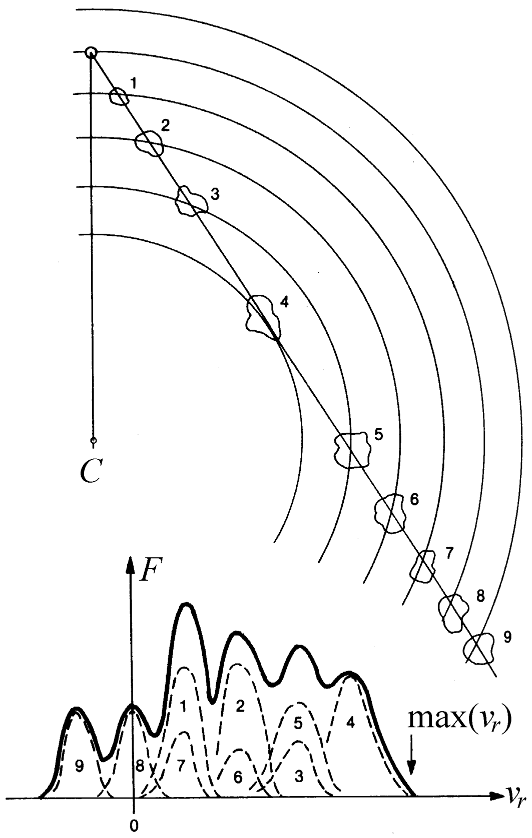

Fig. 29-4 : Typical HI emission spectrum for an observation in the first quadrant of the Galactic Plane showing the contribution of various HI clouds along the line of sight to the overall observed spectrum. The maximum velocity occurs for the HI emission at minimum distance from the Galactic Centre.

4.1 Observational procedure

1. Create the command file shown in Table 4.1 (or edit someone else's file) and save it in a file named *your_initials*`.cmd` in the SRT program folder. You only need to enter the commands in the left column, not the comments in the right column. If the range of Galactic longitudes that you want to observe (usually 270°–360°) are above the horizon as shown in the SRT program's Sky Map, you can start the observations immediately. In this case the first two lines of the command file are not necessary.

   N.B.: We do not recommend making observations in the first quadrant of the Galactic Plane ($\ell = 0°$–$90°$) because $\ell = 90°$ is near the northern limit of the SRT and timing is critical.

2. To initiate your command file, enter the name of the file *your_initials*`.cmd` in the command text-input box at the bottom of the SRT program window and click on the `Rcmdfl` (recall command file) button in the command toolbar at the top right of the SRT program window. If the command is unsuccessful, the text *your_initials*`.cmd file not found` will appear in red in the message board above the text-input box; if this happens check that the command file is in the SRT program folder. If the command is successful the commands in the command file will appear in green in the message board. If you did not have to enter the

command `LST:12:00:00` to wait for $\ell = 270°$ to be high in the sky before beginning your observations, you will be able to start analysing your first set of observations after about 10 minutes. By this time the SRT program will have completed recording data to your first file, e.g. *your_initials*`270.rad`, and have begun recording observations to your second file.

Table 29-2 : SRT command file for observing Doppler-shifted neutral hydrogen in the fourth quadrant of the Galactic Plane. Note there should be no spaces at the beginning of a line, there is no space between '`:`' and '`600`' but there is a space between '`:`' and other commands. The LST start time is optional; if you use it, note that there are neither spaces nor a '`:`' before the Local Sidereal Time command '`LST:xx:00:00`'. **N.B.** Correct typing/spelling is essential

`Commands`	*Comments*
`: ParkPosition`	*/Park telescope until start of observations.*
`LST:12:00:00`	*/Wait until $\ell = 270°$ is at high elevation and*
	/$\ell = 360°$ is well above the horizon.
`: galactic 270 0`	*/Point telescope towards $\ell = 270°$, $b = 0°$.*
`: freq 1419 1`	*/Set frequency away from where HI is expected.*
`: noisecal`	*/Calibrate the receiver using the noise source.*
`: freq 1420.4 50`	*/Set frequency range to cover 1419.4–1421.4 MHz.*
`: record` *your_initials*`270.rad`	*/Start recording to file.*
`:600`	*/Record observations for 600 seconds.*
`: roff`	*/Stop recording observations.*
`: galactic 280 0`	*/Point telescope to Galactic longitude $\ell = 280°$.*
`: freq 1419 1`	*/Set frequency away from where HI is expected.*
`: noisecal`	*/Recalibrate receiver.*
`: freq 1420.4 50`	*/Reset frequency coverage.*
`: record` *your_initials*`280.rad`	*/Record to next file.*
`:600`	
`: roff`	
⋮	*/Repeat the last 7 lines for longitudes up to $\ell = 360°$*
`: ParkPosition`	*/Park telescope at end of observations.*

C2 ▷

4.2 Data Analysis

A detailed account of the data reduction process is given in the Appendix. For each of the datafiles (except $\ell = 360°$), the goal is to obtain the best estimate of the tangential velocity. This involves several steps:

- convert frequencies into Doppler velocities for each ℓ;
- average the recorded signal strength (or system temperature $\overline{T}_{\rm sys}$) at each frequency;
- determine the standard error for each velocity bin;
- estimate the baselevel and subtract it from each bin; and finally,
- determine the most negative velocity for each ℓ and its uncertainty.

We now go through these points in more detail:

1. Open your first observation data file in Excel. The data you need to analyse run from columns G to BD and rows 5 to 38. The first step in producing a plot of mean system temperature ($\overline{T}_{\rm sys}$) versus velocity relative to the LSR is to determine the frequency corresponding to each column of measurements. Copy the start frequency (F38) into G40 and then increment each subsequent column in row 40 by 0.04 (i.e. 40 kHz). This should give you 1421.36 in BD40.

2. You now need to determine the velocity relative to the local standard of rest corresponding to each of these frequencies. Since the component of our velocity along the line of sight relative to $v_{\rm LSR}$ will not change significantly during the ten minute observation period, we can simply use the average value $\overline{v}_{\rm LSR}$ of the entries in column BF. You can then use eqn. (4) to determine the velocities v relative to the LSR for each column of data.

3. We now need to average the recorded temperatures at each frequency. This will give us a much better signal-to-noise ratio for determining the tangential velocities. The best estimate of the error in $\overline{T}_{\rm sys}$ is the standard error s, where $s = \sigma/\sqrt{n}$ and σ is the standard deviation of the n independent temperature measurements. Look for anomalously high values of s; these normally arise from one or two bad measurements which can be eliminated from the spreadsheet and a new value of s determined.

4. With mean system temperatures and standard errors determined the next step is to determine the background noise level and subtract it. Make a scatter plot of $\overline{T}_{\rm sys}$ against v, the velocity relative to $v_{\rm LSR}$, including error bars determined above. Identify the two regions of approximately constant background on either side of the HI emission (eg. regions A and B in Fig. 29-5), average $\overline{T}_{\rm sys}$ over these regions and subtract the mean background $\overline{T}_{\rm bgd}$ from all the $\overline{T}_{\rm sys}$ values. The final plot of relative system temperature versus v will look something like Fig. 29-5.

5. The data reduction process in points 1–4 above needs to be repeated for each value of ℓ. A quick way of doing this is to open the next data file, copy its rows 5 to 38, and paste them over the corresponding rows in the spreadsheet that has all the calculations.

   You now need to determine an accurate value of the most negative HI velocity for each plot (except $\ell = 360°$) and an estimate of its error; these are needed for plotting the rotation curve.

   Explain in your logbook how you arrived at the values you chose.

Question: For $\ell = 360°$ we use the velocity of the peak rather than the tangential velocity. Why?

A tidy way to collect your plots together for your logbook or report is to insert them, one at a time, into a two-column page in Word using copy and paste. Word will automatically resize the chart to fit the column width.

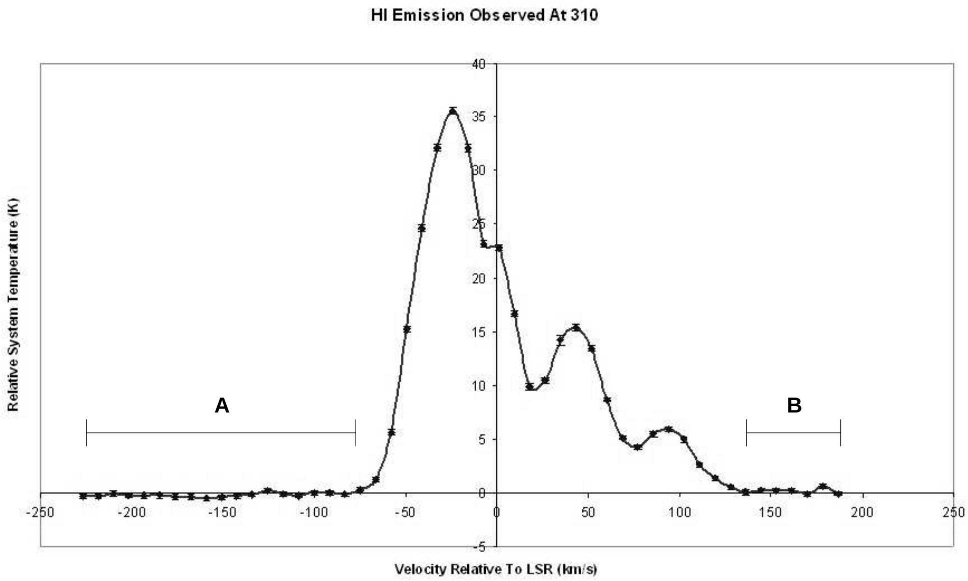

Fig. 29-5 : HI profile at Galactic longitude $\ell = 310°$ with standard errors plotted. The regions marked A and B were used to define the baselevel for this plot. The most negative velocity in this plot was estimated to be $-70 \pm 4\ \mathrm{km\,s^{-1}}$. Figure courtesy of 2005 TSP student, Jian Shen.

$\boxed{\textbf{C3}}$ ▷

4.3 Rotation Curve

1. To determine the rotation curve, create a new spreadsheet with columns for Galactic longitude and maximum (negative) velocity. From these you can calculate the radial distance from the Galactic Centre from eqn. (2) and tangential velocity from eqn. (5), and plot tangential velocity as a function of radius.

2. You will find that the rotational velocity does not depend strongly on distance from the Galactic Centre, at least for $R > 3$ kpc. If the motion was simple Keplerian (i.e. classical circular orbits around a central mass, like the planets in the solar system) the rotational velocity would decrease with an $R^{-1/2}$ dependence. Prove this result.

4.4 Exercises

1. On the basis of Keplerian motion, estimate the enclosed mass, in solar masses ($M_\odot \approx 2 \times 10^{30}$ kg), within the Sun's orbit? Note that the total mass of the Galaxy is around ten times larger than this value, implying most of the mass in the Galaxy lies well beyond the Sun's orbit.

2. What overall Galactic mass distribution with radius fits your data for $R > 3$ kpc? Note that the luminosity of the Galaxy decreases exponentially with radius, so the mass that we infer from the rotation curve cannot be explained by stars, implying the presence of dark matter.

3. Using your rotation curve datapoints, make a plot of enclosed mass as a function of radius from the Galactic centre. Comment.

4. What can you conclude from your observations of the spectrum of the Galactic Centre? Why is the velocity profile so narrow when this is the region where the velocities are greatest?

5. How do you explain the sudden transition from the lack of velocity spread in the HI line at the Galactic Centre to a multi-peaked spectrum at longitudes $\leq 330°$?

C4 ▷

4.4.1 Optional Exercises

1. Observe the HI line emission from the Large and Small Magellanic Clouds. Determine their velocities along the line of sight relative to the LSR.

2. Observe the HI line at several points crossing the Galactic plane. Is the HI emission from a thin disk or can you resolve the thickness of the disk, keeping in mind the resolution of the SRT?

References

[1] B. F. Burke and F. Graham-Smith. *An Introduction to Radio Astronomy*. Cambridge University Press, Cambridge, 1997. [See Chapter 10 Galactic Dynamics, pp. 139–163].

[2] W. B. Burton. The structure of our galaxy derived from observations of neutral hydrogen. In G. L. Verschuur and K. I. Kellermann, editors, *Galactic and Extragalactic Radio Astronomy*, pages 295–358. Springer-Verlag, New York, 1988.

[3] T.L. Wilson K. Rohlfs. *Tools of Radio Astronomy*. Springer-Verlag, New York, 2000. [See Chapter 12 Line Radiation of Neutral Hydrogen].

[4] H. Karttunen, P. Kröger, H. Oja, M. Poutanen, and K. J. Donner. *Fundamental Astronomy*. Springer-Verlag, Berlin, 1997. [See Section 18.3 The Rotation of the Milky Way, pp. 399–406].

A Appendix: manipulating the output data file

These detailed instructions for analysing the SRT data are taken directly from an early version of the Expt 29 notes. They are aimed principally at the novice Excel user. Note that the cell numbering will depend on the exact number of data samples and may differ slightly from one file to the next.

Open your first observation data file, eg. *your_initials*`270.rad`, in Excel and save it as an Excel ".xls" file. It is advisable to save your work often while working through the following procedure.

The format of the output data file is described in the last two pages of the SRT Software Manual. You want to analyse the last block of data in the file (the block starting around row 5 and ending around row 38). The data in this block will have the date and time in column A, the azimuth, elevation and offsets in columns B to E, the start frequency in column F, the measured system temperature of the telescope at each frequency increment in columns G to BD, followed by the computed velocity of the local standard of rest in column BF.

You eventually want to produce a plot of relative averaged measured system temperature versus velocity relative to the local standard of rest. The first step in this process is to determine the frequency corresponding to each column of measurements. A simple way to do this is to enter the formula `=F38` in cell G40 and the formula `=G40+0.04` in cell H40, then copy cell H40 (by, for example, selecting it and pressing Ctrl-C) and paste it into cells I40 to BD40 (by, for example, selecting these cells and pressing Ctrl-V). Row 40 should now contain the frequency of each column of measurements for columns G to BD, with 1419.4 in G40 and 1421.36 in BD40.

Next you want to determine the velocity relative to the local standard of rest corresponding to each of these frequencies. Since the component of our velocity along the line of sight of the observation relative to the local standard of rest $v_{\rm LSR}$ will not change much during the ten minute observation period, it is reasonable to use the average of this value $\bar{v}_{\rm LSR}$ in your calculations. You can compute this by entering, for example, `=AVERAGE(BF5:BF38)` in cell BF40. Now, in cells G41 to BD41 use (4) to determine the velocity relative to the local standard of rest corresponding to each frequency in cells G40 to BD40. The units of $\bar{v}_{\rm LSR}$ in cell BF40 are km/s. Note that in your formula when referring to $\bar{v}_{\rm LSR}$, ensure you make an absolute reference (i.e. `$BF$40`) rather than a relative reference (i.e. `BF40`), to ensure you always refer to cell BF40 when you copy your formula for cell G41 to cells H41 to BD41 (otherwise your copied formula will refer to cells BG40 to DC40 rather than to cell BF40 and your calculations will be incorrect).

Averaging your measured temperature results will improve the ratio of the received hydrogen emission signal relative to the noise introduced in taking the measurements. As long as the noise is random, the improvement in the signal to noise ratio is proportional to the square root of the number of independent measurements. In cell G42 enter the formula `=AVERAGE(G5:G38)` to calculate the average system temperature for the column. Copy and paste this formula into cells H42 to BD42. To determine relative system temperatures first determine the minimum average system temperature by entering the formula `=MIN(G42:BD42)` in cell G43. Now enter the formula `=G42-$G$43` in cell G44 and copy and paste this formula into cells H44 to BD44. In cell G45 enter the formula `=STDEV(G5:G38)/SQRT(33)` to determine the standard error in the system temperatures for the column. Copy and paste this formula into cells H45 to BD45. This will be used next in determining the error bars for these measurements.

To help clarify what's where you may like to enter the text `freq` in F40, `vel` in F41, `ave temp` in F42, `min temp` in F43, `rel temp` in F44 and `std err` in F45.

Now plot the relative averaged measured system temperature versus the velocity relative to the local standard of rest. One way to do this is to use the Chart Wizard: (i) select “Chart...” from the “Insert” menu, (ii) select the “XY Scatter” Chart type and the “Scatter with data points connected by smoothed lines” Chart sub-type and click “Next >”, (iii) select cells G41 to BD41 press the comma key (“,”) and select cells G44 to BD44 then click “Next >”, (iv) enter a title for the graph such as “HI emission observed in direction l=0, b=0”, an x-axis title such as “Velocity relative to the Local Standard of Rest (km/s)”, a y-axis title such as “Relative System Temperature (K)”, remove the gridlines and the legend and click “Next >”, (v) choose Place Chart as a new sheet and click “Finish”, (vi) double-click on the chart line to open the “Format Data Series” window, click the “Y Error Bars” tab, click the “Custom” error amount, click the tab next to the “Chart1” tab in the bottom left of the Excel window to show the spread-sheet data values and select cells G45 to BD45, repeat this procedure to enter the same values for the minus (“−”) error values and click “OK”, (vii) double-click on the y-axis to open the “Format Axis” window, click on the “Scale” tab, enter “0” as the minimum and click “OK”, (viii) double-click on the gray plot background to open the “Format Plot Area” window, click “None” under “Area” and click “OK”.

A tidy way to collect your plots together is to insert them into a two-column page in Word. Copy your chart by clicking outside the boxed plot area and pressing Ctrl-C (or select “Copy” from the “Edit” menu). Open a new document in Word, select “Page Setup” from Word’s “File” menu and set the top, bottom, left and right margins to 1 cm and click “OK”, select “Columns...” from the “Format” menu, select “Two” from the “Presets” options and click “OK”. Now paste (Ctrl-V) your chart into the new document — Word will automatically resize the chart to fit the column width. Next a quick method is given to produce plots for the rest of your data files. Each time you make another plot you can add it to the plots in the Word document. When you have the complete set of plots, print them out and put a copy in your logbook.